Paper : Differential Equations

Section–A

1. i) Define Ordinary differential equation.

ii) Define (a) Explicit solution (b) Implicit Solution (c) singular solution (d) General solution, (e) Particular Solution.

iii) What is IVP? Also define BVP.?

iv) Verify $y = ce^{mx}$ is a solution of $y' = my$?

v) What is the order and degree of the given differential equation?

a) $\left(\frac{dy}{dt}\right)^3 = \sqrt{\left(\frac{dy}{dt}\right)^2 + 1}$ (b) $x^2dy + y^2dx = 0$ c) $y'' + yy' = x$

(d) $y' + xy = e^x$ (e) $y^{iv} + 3(y')^3 + y = 0$

vi) Define Homogeneous function of degree 'n'.

vii) How can we test the Exactness of a differential equation of Ist order and Ist degree?

viii) Define Integrating factor?

ix) If M(x,y) dx + N (x,y) dy = 0 is an homogeneous function then what is the integrating factor if $Mx+Ny \neq 0$?

x) If M(x,y) dx + N (x,y) dy = 0 can be written as $f_1(xy)\, ydx + f_2(xy)\, xdy = 0$ and $Mx–Ny \neq 0$ then what is the integrating factor?

xi) If $\frac{\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(y)$ then what is the integrating factor ?

xii) If $\frac{\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}}{M} = f(y)$ then what is the integrating factor?

xiii) What is the general form of Linear differential equation?

xiv) Write the integrating factor of $(1+x)\frac{dy}{dx} – xy = 1–x$

xv) Write the integrating factor of $(1+y^2)\, dx = (\tan^{-1}y – x)\, dy$

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xvi) Write the integrating factor of $y\,dx + (xy^2+x-y)\,dy = 0$

xvii) Write the general form of Bernoulli Equation?

xviii) Reduce the equation $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x}\log x$ to linear differential equation.

xix) Reduce the equation $x\frac{dy}{dx} + y = x^4y^3$ to linear differential equation.

xx) What is Malthusian law of papulation growth?

xxi) What is orthogonal trajectory of the given family?

xxii) Write general linear equation of order n?

xxiii) What is homogeous linear equation?

xxiv) Write the auxiliary equation of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6 = 0$

xxv) If $m_1, m_2, m_3, \ldots, m_n$ are the n distinct roots of homogeneous linear equation with constant coefficients, then what is the general solution?

xxvi) If two roots m_1 and m_2 of the auxiliary equation are equal then write the solution?

xxvii) If the auxiliary equation has the conjugate complex roots, $\alpha+i\beta$ and $\alpha-i\beta$ then write the solution?

xxviii) Write the auxiliary equation of $y^{iv} - 4y''' + 8y'' - 8y' + 4y = 0$?

xxix) Using symbolic operation write the result of $\frac{e^{ax}}{F(D)}$?

xxx) For any constant a, using symbolic operation write the value of $\frac{1}{F(D)}[e^{ax}v(x)]$?

xxxi) Write the value of $\frac{1}{F(D^2)}\sin ax$?

xxxii) When we can say a function is an undetermined coefficient?

xxxiv) Write the expression for the Wronskian $w(y_1, y_2)$

xxxv) What is the general form of Euler equation?

xxxvi) If 2 and –2 are the distinct real roots of an Euler equation then write the general solution?

xxxvii) What do you mean by equilibirium points?

Section–B

2. i) Verify $y = \sqrt{16 - x^2}$ is a solution of $x+yy' = 0$?

ii) Find a differential euqatioin by using the function $r = a(1-\cos\theta)$?

iii) Solve the IVP $\frac{dy}{dx} + y = 2xe^{-x}$, $y(1) = 0$, given the general solution as $y = (x^2+c)e^{-x}$?

iv) Solve the IVP $y'' + 5y' + 4y = 0, y(0) = 0, y'(0) = 1$ Given that the general solution is $y = c_1e^{-4x} + c_2e^{-x}$?

v) Verify $y = \frac{1}{1+x^2}$ is a solution of $(1+x^2)y'' + 4xy' + 2y = 0$?

vi) Find the values of m such that the function 'f' defined by $f(x) = e^{mx}$ is a solution of the differential equation $y''+4y = 0$?

vii) Verify $y = x \tan x$ is a solution of $xy' = y+x^2+y^2$?

viii) Solve $\frac{dy}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} = 0$?

ix) Solve $\frac{dy}{dx} = \frac{2x}{\cos y}$?

x) Solve $y\frac{dy}{dx} = \cos^2 wx$.

xi) Solve $x\frac{dy}{dx} + y^2 = 9$

xii) Reduce the equation $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$ to homogeneous function?

xiii) Solve $\frac{dy}{dx} = \frac{x+y+4}{x+y+6}$

xiv) Solve the IVP, $y' \tan x = y+1, y(2) = 0$.

xv) Test the exactness of the differential equation $(y\cos x + \sin y + y)\,dx + (\sin x + x\cos y + x)\,dy = 0$?

xvi) Test the equation $e^y\,dx + (xe^y + 2y)\,dy = 0$ for exactness?

xvii) Test the exactness of the equate $(xe^{xy} + 2y)\,dy + ye^{xy}dx = 0$?

xviii) Find the integrating factor of the equation, $(x^2y - 2xy^2)\,dx + (3x^2y - x^3)\,dy = 0$?

xix) Find the integrating factor of $(x^2y^2 + xy + 1)\,y\,dx + (x^2y^2 - xy + 1)\,xdy = 0$?

xx) Find the integrating factor of $(x^2 + y^2)\,dx - 2xydy = 0$

xxi) Find the integrating factor of $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$?

P.T.O.

xxii) Find I.F for $x\frac{dy}{dx} - (x+1)\, y = x^2 - x^3$?

xxiii) Find I.F for $(1+y^2)\, dx = (\tan^{-1}y - x)\, dy$?

xxiv) Find I.F for $ydx + (xy^2 + x - y)\, dy = 0$

xxv) Find I.F for $x^2\frac{dy}{dx} + 2xy - x + 1 = 0$

xxvi) Reduce the equation $x\frac{dy}{dx} + y = x^4y^3$ to linear form?

xxvii) Reduce the equation $\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$ to liner form?

xxviii) Reduce to linear form of $\frac{dy}{dx}(x^2y^3 + xy) = 1$?

xxix) Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6 = 0$

xxx) Solve $\frac{d^2y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$

xxxi) Solve the IVP, $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$, $x(0) = 2$, $x'(0) = 0$

xxxii) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ with $y(0) = 1$, $y'(0) = 0$

xxxiii) Find the general solution of $y'' - 4y' + 4y = 0$

xxxiv) Find the general solution of $y'' + 8y = 0$

xxxv) Find the general solution of $y^{iv} + 2y'' + y = 0$

xxxvi) Solve $(D^2 - 2D + 1)\, y = e^{-x}$.

xxxvii) Find the particular integral of $(D-2)^2\, y = e^{2x}$?

xxxviii) Find the P.I. of $(D^2 + 4)\, y = x \sin x$.

xxxix) Solve $(D^2+1)\, y = xe^{2x}$.

xxxx) Find particular solution of $(D^2+1)\, y = \sec x$.

Section – C

3. i) Solve $(xy+2x+y+2)\, dx + (x^2+2x)dy = 0$

ii) Solve $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$

iii) Solve $\frac{dy}{dx} = \frac{x+y+4}{x+y-6}$

iv) Solve $(2x+3y+1)\, dx + (4x+6y+1)\, dy = 0$, $y(-2) = 2$

v) Solve the IVP, $xy' = (y-x)^3+y$, $y(0) = 0$.

vi) Solve $xe^{x^2+y^2}\, dx + y\ (e^{x^2+y^2}+1)\, dy = 0$

vii) Solve the IVP. $(3x^2y+8xy^2)\, dx + (x^3+8x^2y+12y^2)\, dy = 0$, $y(2)=1$.

viii) Solve the IVP. $(2x\cos y + 3x^2y)\, dx + (x^3 - x^2\sin y - y)\, dy = 0$, $y(0) = 2$.

ix) Solve $(3-y) / x^2 dx + (y^2-2x) / xy^2\, dy = 0$, $y(-1) = 2$.

x) Solve $y(xy+2x^2y^2)\, dx + x\,(xy-x^2y^2)\, dy = 0$

xi) Solve $(x^2+y^2+2x)\, dx + 2ydy = 0$

xii) Solve $(xy^3+y)\, dx + 2\,(x^2y^2+x+y^4)\, dy = 0$

xiii) Solve $x\cos y\, dy - \sin y\, dx = 0$

P.T.O.

xiv) Solve $xdy + y\,dx + 3x^3y^4dy = 0$

xv) Solve $(\sin x + \cos x \tan y)(dx+dy) + 2\sin y\,dy = 0$

xvi) Solve $x\frac{dy}{dx} - (x+1)y = x^2 - x^3$.

xvii) Solve the IVP, $\frac{dy}{dx} + y\tan x = \sin 2x$, $y(0) = 1$.

xviii) Solve $(1+x)\frac{dy}{dx} - xy = 1 - x$.

xix) Solve $y\,dx + (xy^2 + x - y)\,dy = 0$

xx) Solve the IVP, $x^2\frac{dy}{dx} + 2xy - x + 1 = 0$, $y(1) = 0$

xxi) Solve the IVP, $\frac{dy}{dx} + \frac{y}{x} = \frac{\sin x}{x}$, $y(0) = 0$

xxii) Solve $\frac{dy}{dx} + 7y = 2x + 9$, $y(1) = 1$.

xxiii) Solve $\frac{dy}{dx} + y\cos x = \sin 2x$, $y(\pi) = 0$

xxiv) Solve $e^x[y - 3(e^x+1)^2]\,dx + (e^x+1)\,dy = 0$, $y(0) = 4$.

xxv) Solve, $\frac{dy}{dx} + y = f(x)$ $\begin{cases} = e^{-x},\ 0 \le x < 2,\ y(0) = 1 \\ e^{-2},\ x \ge 2 \end{cases}$

xxvi) Solve $(x+2)\frac{dy}{dx} + y = f(x)$, $f(x) = \begin{cases} 2x,\ 0 \le x \le 2,\ y(0) = 4 \\ 4, \quad x > 2 \end{cases}$

xxvii) Solve $2x^2\frac{dy}{dx} = (x-1)(y^2 - x^2) + 2xy$

xxviii) Solve $3y^2\frac{dy}{dx} + xy^3 = x$

xxix) Solve $\frac{dy}{dx} + xy = x^3y^3$

xxx) The rate of decay of radioactive material is proportional to the amount present. The half life period of a sample of the material is 10 years. How long does it take for 90% of the original amount of the material to disintegrate?

xxxi) Find the current in the simple circuit with $C = \infty$ and $E(t) = E_0 \sin wt$.

xxxii) If the half-life of radium is 1600 years, how long will it take a mass radium to disintegrate until but 10 percent remains?

xxxiii) Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$, λ being the parammeter.

xxxiv) Solve the IVP, $xy\frac{dy}{dx} = (y-1)(x+1)$, $y(-\frac{1}{2}) = -\frac{1}{2}$

xxxv) Solve $y(x^2 - xy+y^2) - x\frac{dy}{dx}(x^2+xy+y^2) = 0$, $y(1) = 1$

xxxvi) Solve $\cos^2 x \frac{dy}{dx} = \frac{y}{y+1}$, $y(0) = 1$

xxxvii) Solve $y^{IV} - 2y^{III} + y^{II} = 0$

xxxviii) Solve $y^{IV} - 5y^{III} + 6y^{II} + 4y^{I} - 8y = 0$

xxxix) Solve $9y'' + 6y' + 5y = 0$, $y(0) = 6$, $y'(0) = 0$

xxxx) Solve $y''' - 6y'' + 11y' - 6y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 2$.

xxxxi) Find the PI of $(D^3+1)y = e^x \cos x + \sin 3x$?

xxxxii) Solve $(D^2+4)y = 4\tan 2x$

xxxxiii) Solve by the method of undetermined coefficients. $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 5y = 5\sin 2x + 10x^2 + 3x + 7$.

P.T.O.

xxxxiv) Find a particular solution of the following equation by the method of variation of parameter. $(D^2+4)y = \sec^2 2x$.

xxxxv) Find a particular solution by the method of variation of Parameter: $(D^2 + 4D + 5)\, y = e^{-2x} \sec x$.

xxxxvi) Solve $x^2\frac{d^2y}{dx^3} - x\frac{dy}{dx} - 3y = x^2 \log x$.

xxxxvii) Solve $(x^2D^2 + 4xD + 2)\, y = \sin x$.

xxxxviii) Solve $(2x^2D^2 + 5xD + 1)\, y = ax + b$.

xxxxix) Solve $(x^3D^3 + 2x^2D^2 - xD + 1)\, y = x^2 \log x$.

xxxxx) Solve $x^2y'' - 3xy' - 5y = 0$

xxxxxi) Solve $x^2y'' - 2xy' + 2y = 2x^3$; $y(\pm 1) = 0$

xxxxxii) Solve $x^2y'' - 2xy' + 2y = x$; $y(1) = 4$, $y'(I) = 0$

xxxxxiii) Solve $x^2y'' - xy' + y = \log x$; $y(1) = 3$, $y'(I) = 0$

xxxxxiv) Solve IVP, $(D^2 + 2D + 5)\, y = 0$; $y(0) = 0$, $Dy(0) = 1$.

xxxxxv) Find the general solution of $(D^2 + D)\, y = x \sin^2 x$.

xxxxxvi) Solve the IVP, $(D^2 + D - 2)\, y = 0$, $y(0) = 3$, $Dy(0) = 0$.

xxxxxvii) Solve the IVP, $3y'' + 4y' + y = e^{-t} \sin t$; $y(0) = 1$, $y'(0) = 0$.

xxxxxviii) Solve $4y'' + 4y' + y = \cos\frac{x}{2}$; $y(0) = 0$, $y'(0) = 0$

xxxxxix) Find the integral curve of the differential equation $y'' - y = x^2 + 4e^x$, Which touches the x-axis. at the origin?

xxxxxx) Solve $y'' - 3y' + 2y = -5 + 12e^{-x}$; $y(0) = \frac{7}{2}$; $y'(0) = 9$.

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